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SHORTER COMMUNICATION

BOUNDARY-LAYER FLOW WITH TRANSPIRATION ON AN ISOTHERMAL FLAT PLATE

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NOMENCLATURE

B, $(v_0/u_\infty)Re^{1/2}$;
c_p, isobaric specific heat-capacity;
c_f, $\tau/(1/2)\rho u_\infty^2$;
k, thermal conductivity;
Nu, $Qx/(T_0 - T_\infty)$;
Pr, $\mu c_p/k$;
Q, local surface (outward) heat flux;
Re, $u_\infty \rho x/\mu$;
T_0, surface temperature;
T_∞, free-stream temperature;
u_∞, free-stream velocity;
v_0, local normal surface (outward) velocity;
x, streamwise distance from leading edge.

Greek symbols
ζ, defined in equation (5);
μ, viscosity;
ξ, $NuRe^{-1/2}$;
ρ, density;
τ, local surface shear stress.

FOR uniform-property, steady, laminar, boundary-layer flow with negligible dissipation and uniform free-stream velocity and temperature, similar solutions to the momentum and energy equations may be obtained provided that the normal velocity at the surface varies as $x^{-1/2}$. Numerically-obtained results, giving the relationship between the surface heat- and mass-transfer parameters, $NuRe^{-1/2}$ and $(v_0/u_\infty)Re^{1/2}$, for various values of Pr, have been obtained and are tabulated by Kays [1]. The present note provides approximate formulae for this relationship. The equations given are valid for zero and infinite suction (for all Pr) and are in close agreement with the available numerical results for various Pr. Since the numerical results are for values of $(v_0/u_\infty)Re^{1/2}$ ranging from effectively large negative values (strong suction) to "near-separation" positive values, the formulae given should be accurate for the whole range of values of the transpiration parameter for Prandtl numbers in the range covered by the numerical solutions i.e. $0.55 < Pr < 1$. Further, since the formulae are correctly anchored at $(v_0/u_\infty)Re^{1/2} = 0$ and for $(v_0/u_\infty)Re^{1/2} \to -\infty$, they should be suitable for extrapolation to Prandtl numbers outside the above range, particularly for numerically small values of the transpiration parameter and for strong suction.

For zero transpiration the following result, given in [2],

$$NuRe^{-1/2} = Pr^{1/2}(27.8 + 75.9Pr^{0.306} + 657Pr)^{-1/6}, \quad (1)$$

agrees with numerical results [3] to within 0.33% over the range $0.0001 < Pr < 20000$.

For infinite suction ($v_0 \to -\infty$) it may be seen from [4] that:

$$NuRe^{-1/2} \to -\left(\frac{v_0}{u_\infty}\right)Re^{1/2}Pr. \quad (2)$$

In the first instance attention was restricted to "suction" ($v_0 < 0$) where the results are of interest for the related condensation problem [5]. Various equations, having the limiting behaviour indicated by equations (1) and (2), were

Table 1.*

$B = \left(\frac{v_0}{u_\infty}\right)Re^{1/2}$	$\xi = NuRe^{-1/2}$, $Pr = 0.55$: Numerical solution	$Pr = 0.55$: Eqn. (3)	$Pr = 0.55$: Eqn. (8)	$Pr = 0.7$: Numerical solution	$Pr = 0.7$: Eqn. (3)	$Pr = 0.7$: Eqn. (8)	$Pr = 0.8$: Numerical solution	$Pr = 0.8$: Eqn. (3)	$Pr = 0.8$: Eqn. (8)	$Pr = 1.0$: Numerical solution	$Pr = 1.0$: Eqn. (3)	$Pr = 1.0$: Eqn. (8)
−5.000	2.811	2.811	2.813							5.049	5.048	5.047
−2.500	1.481	1.481	1.484	1.850	1.850	1.850	2.097	2.096	2.096	2.590	2.590	2.591
−0.750	0.605	0.605	0.606	0.722	0.721	0.719	0.797	0.797	0.794	0.945	0.947	0.945
−0.250	0.377	0.378	0.377	0.429	0.431	0.427	0.461	0.464	0.459	0.523	0.527	0.522
0				0.292	0.292	0.292	0.307	0.306	0.306	0.332	0.331	0.331
0.250				0.166		0.166	0.166		0.167	0.165		0.165
0.375				0.107		0.108	0.103		0.104	0.094		0.094
0.500				0.052		0.052	0.046		0.046	0.036		0.034

*Comparison of equation (3) (with $a = 0.941$, $b = 1.14$, $c = 0.93$) and equation (8) (with $a = 1.57$, $b = 1.27$, $c = 1.76$, $d = 0.84$) with numerical solutions. (For $Pr = 0.7$, 0.8, 1.0 the numerical results are taken from Kays [1]. For $Pr = 0.55$ the numerical results were obtained from those given by Sparrow *et al.* [6] for the identical diffusion problem with $Sc = 0.55$.)

considered and used to fit the numerically-obtained results. The form finally adopted was:

$$\xi = \zeta(1+a(-B)^b Pr^c)^{-1} - BPr, \quad (3)$$

where

$$\xi = NuRe^{-1/2} \quad (4)$$

$$\zeta = Pr^{1/2}(27.8+75.9Pr^{0.306}+657Pr)^{-1/6} \quad (5)$$

$$B = (v_0/u_\infty)Re^{1/2}. \quad (6)$$

The values of a, b and c were found by minimization of the sum of squares of residuals of ξ using the data ($B < 0$) given in Table 1. To avoid retaining excess digits the constants were found in stages, after each of which one constant was rounded and fixed and the remaining constants redetermined. The final values were $a = 0.941$, $b = 1.14$, and $c = 0.93$. As may be seen from Table 1, equation (3), with the above constants, is in very close agreement with the numerical solutions.

So as to include the "blowing" data ($B > 0$), while continuing to satisfy equations (1) for $B = 0$ and (2) for $B \to -\infty$, equation (3) was modified to:

$$\xi = \left(\frac{1+ad^b Pr^c}{1+a(d-B)^b Pr^c}\right)\zeta - BPr. \quad (7)$$

The four constants were found as described above using the numerically-obtained results for both "blowing" and "suction". The values obtained were $a = 1.57$, $b = 1.27$, $c = 1.76$, and $d = 0.84$, so that equation (7) becomes:

$$\xi = \left(\frac{1+1.26Pr^c}{1+a(d-B)^b Pr^c}\right)\zeta - BPr. \quad (8)$$

The close agreement between equation (8) and the numerical results may be seen from Table 1.

Finally, since for $Pr = 1$, the appropriately non-dimensionalized momentum and energy equations, and their boundary conditions, are identical, the relationship between the surface shear stress and the mass-transfer parameter may be obtained directly from equation (8) by replacing ξ by $(c_f/2)Re^{1/2}$, thus:

$$\frac{c_f Re^{1/2}}{2} = \frac{0.747}{1+1.57(0.84-B)^{1.27}} - B. \quad (9)$$

The data values of ξ and those given by equation (8) in the $Pr = 1$ column of Table 1 are the same as the values of $(c_f/2)Re^{1/2}$ given by numerical solution and by equation (9) respectively.

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